Preparing For The SAT
And Other Aptitude Tests

For the ATARI® 400/800™ computer

If you can't load the first program on this cassette, follow these directions exactly.

Rewind the cassette all the way. Set the cassette counter to 000. Now fast forward the cassette to 002 or 003 on the counter. No farther. **CLOAD** from this point.

Do not return cassette to PDI without return authorization.

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And Other Aptitude Tests

For the ATARI ® 400/800™ computer

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INTRODUCTION

Many people who have above-average intelligence do poorly on IQ tests and aptitude tests, such as the Scholastic Aptitude Test (SAT). One reason is that these individuals have not developed problem-solving skills needed to answer the types of questions found on these tests. The test taker must be able to analyze a question, separate it into its component parts, and then test a variety of answers to find the one that best fits the problem. The poor problem-solver does not follow any real plan of action. Instead, he or she looks for an instant answer or writes the first solution that comes to mind. On a multiple-choice test, the poor problem-solver usually jumps at the first answer that looks as though it might be correct. In short, he or she panics and is unable to think through the problem at hand.

Preparing for the SAT helps you to develop the problem-solving skills needed to do well on IQ and aptitude tests. The initial course in the series describes ways to improve your test-taking skills. Each of the remaining courses in the series contains a group of lessons that help you with a specific category of questions. All of the courses will help you to develop your skills so you can handle even the most complex questions.

MATERIALS

Preparing for the SAT consists of the following materials:

- *Taking Aptitude Tests* This course has four functions. It explains the purpose of standardized IQ and aptitude tests. It discusses some of the false beliefs surrounding such tests. It describes ways to improve your test-taking skills. And it presents a strategy for answering those questions that are most likely to pay off with correct answers.
- *Vocabulary Builder* These two courses are designed to help develop your vocabulary and to build skills needed to answer synonym and antonym questions.
- *Analogies* This course describes the common types of analogies and provides practice in analyzing and solving analogy problems.
- *Number Series* This course teaches you how to analyze number series patterns and provides practice in number series problems.
- *Quantitative Comparisons* This course reviews mathematics, from elementary arithmetic through algebra and plane geometry, and provides practice in solving the types of mathematics problems found on standardized tests.
- *Making the Grade* This booklet, written by PDI's President, John Victor, gives you a better idea of what testing is all about, presents several important test-taking strategies, and describes the various types of questions you are apt to encounter on IQ and aptitude tests.

HOW TO USE THE PROGRAM

We recommend that you begin by loading Side A of the cassette *Taking Aptitude Tests* into your computer. Then read Chapters 1 and 2 in the booklet *Making the Grade*. This will give you an excellent introduction to standardized tests. Also skim the rest of the booklet so you know what other information it contains. Next, load Side B of the same cassette. It requires you to take a sample test. A good source of such tests is *Barron's How to Prepare for College Entrance Examinations* (available in bookstores and libraries).

After completing the cassette, read Chapter 5 of *Making the Grade*. This will help you improve your problem-solving skills.

Now you are ready for the remaining courses. These do not have to be used in any particular order. However, always begin with the introduction or first lesson on a cassette or disk, even if you think you know enough to skip ahead. Do the lessons in order because each builds on the one before. Repeat each lesson until you feel you can handle the examples in that lesson.

Follow directions in your computer manual to load each lesson. You can make each session last as long as you wish. Each lesson can be completed in about 30 minutes. If you want to stop for more than a few minutes, it is best to turn off the computer and TV monitor.

With the exception of *Number Series*, which is self-testing, there is a test at the end of each course. You can take the test after completing all the lessons or you can take it earlier, to see how well you can handle sample test questions.

Making the Grade contains chapters on most of the types of questions found on standardized tests. Read these chapters to increase your familiarity with typical test questions and to improve your ability to answer various types of questions.

HOW TO USE VOCABULARY BUILDER

Most tests that measure a person's ability to use words include a section on word meanings. Tests such as the Scholastic Aptitude Test and other college entrance examinations have a large part of the test devoted to questions on word meanings.

There are several ways to test a person's understanding of word meanings. *Vocabulary Builder* covers the two most common types of questions. The first type is the synonym question, in which you must find a word choice that means the *same* as a given word. For example:

quick: (1) fast (2) hard (3) silent
(4) dishonest (5) quiet

Answer: (1)

The second type deals with antonyms—words of opposite meaning. Here you must pick a word choice *opposite* in meaning from the given word. For example:

quick: (1) fast (2) slow (3) silent
(4) dishonest (5) loose

Answer: (2)

The tendency when answering antonym questions is to forget that the correct answer is opposite in meaning. Don't mistakenly choose the word that has the same meaning.

Organization of the Course

Vocabulary Builder is divided into Parts I and II, each on its own cassette or disk. The vocabulary questions in Part II are significantly more difficult than those in Part I. Otherwise, the organization of the two sections is the same.

> Cassette version: In each course, Side A of the cassette contains five lessons of synonym questions. Side B contains five antonym lessons, plus a vocabulary test.

Each lesson gives you the opportunity to work with as many or as few problems as you wish. You can start anywhere from Question 1 to 40. You could do Questions 1 through 10 on one day and 11 through 40 on another day.

A single pass through a lesson may not be enough. You might have to take a lesson two or even three times to get the feel of the questions.

If you miss a question, you are given several more chances to choose the correct answer. However, don't guess at random! If you miss a question twice, look up the meaning of the words in a dictionary before attempting the question again.

Also, read Chapter 6 in *Making the Grade*. It explains the importance of word families and tells you how to figure out the meanings of words.

HOW TO USE ANALOGIES

An analogy can be one of the toughest of verbal questions to handle because all of the meanings, connotations and uses of words have to be considered when answering the question. Here is a sample question:

BIG is to LITTLE as:
(1) man is to men
(2) dog is to cat
(3) run is to walk
(4) hot is to cold

A test taker must analyze the relationship between the words "big" and "little" and find the word pair that has the same relationship. Big and little are opposite extremes. Therefore, the correct choice must also show opposite extremes. The best choice is (4)—hot is to cold. (3) might also be thought of as a sort of opposite. The key is the phrase "best choice." (4) is more of an opposite than (3).

Organization of the Course

The course consists of six lessons, plus a final test of your skill with analogies. Each lesson gives you the opportunity to work with as many or as few problems as you wish. For example, in Lesson 1 you can do anywhere from 1 to 30 problems at one sitting. You can start a lesson at any point you wish from problem 1 to problem 30. You can do problems 1 through 10 on one day and then do 10 through 30 on the next. You may also find certain problems that you must do several times before getting the "feel" of the problems.

> Cassette version: Side A of the tape cassette contains Lessons 1 to 4. Side B contains Lessons 5 and 6, plus the final test.

Lessons 1 And 2

In these lessons the student must classify word pairs. This may prove to be a bit more difficult than some analogies problems themselves, but the practice will provide you with the ability to analyze almost any type of analogies problem. Here is a sample:

WRITER is to TYPEWRITER is what type of analogy?
(1) worker to object created
(2) person to goal
(3) worker to tool
(4) tool to object created
(5) cause and effect

The correct answer is (3)—worker to tool.

Each of these lessons consists of 30 problems. An evaluation of your performance is given at the end of each session. It is recommended that you repeat the lesson if you score below 60% correct, but you might also want to review the problems until you can easily score 90%.

Lesson 3

Lesson 3 consists of 20 problems in the following form:

MOTHER is to DAUGHTER as FATHER is to:
(1) boy
(2) son
(3) uncle
(4) child
(5) sibling

The answer to the above problem is (2)—son.

If a mistake is made the program will give you a hint by first telling you what kind of analogy is shown in the problem. The program will give you a second hint if you wish. If you correctly answer the question on the second try, your correct response is worth only .7 of a correct answer. A second error will cause the program to give you the correct answer.

Lessons 4 and 5

Lessons 4 and 5 each consist of 15 questions of the following type:

KENNEL is to BULL TERRIER as:
(1) dog is to cat
(2) cage is to bird
(3) cage is to green parrot
(4) house is to person

The correct answer is (4)—cage is to green parrot. The above problem is specific about the breed of dog kept in the kennel, so the correct matching word pair must also be specific about the breed of bird kept in a cage. Except for the form of the questions, these lessons work like Lesson 3.

Lesson 6

This lesson is similar to the previous lessons except that the problems are presented in a different way.

EROSION: WATER : :
(1) ocean : wind
(2) fog : travel
(3) solid : liquid
(4) aging : time
(5) melt : heat

The answer is (5)—melt : heat.

Types Of Analogies

One way to analyze analogies is to reduce the relationship between the two words to a simple sentence. Here are some examples:

Painter is to **Brush**	A uses B in his or her work.
Diamond is to **Gem**	A is an example of B.
Literate is to **Read**	One who is A can B.
Illness is to **Fever**	A can cause B.
Pauper is to **Money**	A does not have B.
Blinders is to **Vision**	A interferes with B.
Police is to **Criminals**	A protects us from B.
Foundation is to **Building**	A supports B.
Sprint is to **Run**	A is an intensified form of B.

The above system is a good way to start to analyze word relationships, but you must do more. For example, how would you analyze DOG is to CAT? A chases B? A is bigger than B? A and B are both pets? Any of these may be correct.

Another, more precise way to analyze analogies is to classify the word pairs. Here are 27 basic types of analogies:

Type 1—part to whole
Examples: leg is to person
wheel is to car
lace is to shoe

Type 2—type to one of its characteristics
Examples: skunk is to bad smell
elephant is to large
old car is to rusty

Type 3—things that are part of the same object
Examples: ear is to eye (part of head)
halfback is to center (part of a football team)
branch is to leaf (part of a tree)

Type 4—measurement to what is measured
Examples: pint to liquid
meter to distance
decibel to sound

Type 5—measurement to object measured
Examples: pint to beer
meter to cloth
decibel to radio speaker

Type 6—class to species
Examples: dog to greyhound
insect to fly
vehicle to truck

Type 7—group to member
Examples: army to sergeant
pack to wolf
team to player

Type 8—things in same class
Examples: truck to car (both vehicles)
boxer to runner (both athletes)
crow to robin

Type 9—things with a feature in common
Examples: match to lightbulb (both give off light)
clock to car (both have gears)
bottle to lens (both made of glass)

Type 10—measures of the same thing
Examples: pint to gallon (measures of liquid)
pound to kilogram (measures of weight)
goals to touchdowns (measures of scores)

Type 11—hierarchies
Examples: general to private
parent to child
manager to salesperson

Type 12—cause and effect
Examples: hit to break
fire to burn
switch on to operate

Type 13—things to what they do
Examples: cork to plug up
soap to clean
pencil to write

Type 14—tools to material
Examples: saw to wood
hammer to nail
sewing machine to cloth

Type 15—tools to what they create
Examples: potter's wheel to vase
motor to power
saw to cabinet

Type 16—condition to what happens in that condition
Examples: storm to rain
sick to fever
happy to smile

Type 17—worker to object created
Examples: carpenter to cabinet
seamstress to dress
assembly worker to car

Type 18—worker to tool
Examples: carpenter to hammer
mechanic to wrench
surgeon to scalpel

Type 19—person to goal
Examples: general to victory
runner to 4-minute mile
climber to mountain peak

Type 20—person to something he or she avoids
Examples: child to the dark
claustrophobic to a closed space
general to defeat

Type 21—synonyms and antonyms
Examples: dishonest to unethical (synonym)
strong to weak (antonym)

Type 22—things that go together
Examples: cup to saucer
clouds to rain
driver to car

Type 23—a thing dependent on another thing
Examples: person to food
fire to fuel
health to clean air

Type 24—a thing derived from another thing
Examples: metal to ores
cell growth to protein
cabinet to wood

Type 25—a specific condition that occurs on/to a particular thing
Examples: wind to atmosphere
rash to skin
vibrate to violin string

Type 26—opposing things or forces
Examples: electron to proton
ying to yang
Republicans to Democrats

Type 27—words related by grammar
Examples: horse to sky (both nouns)
blue to large (both adjectives)

Of course, some of these classifications overlap. Some word pairs will fit into several classifications. The idea is to pick the class that *best* fits the relationship.

Once you can categorize the relationships between word pairs, you can eliminate choices. However, when answering analogy-type questions, there may be more than one relationship involved, as shown in this example:

SELDOM is to FREQUENTLY as:
(1) occasionally is to rarely
(2) top is to bottom
(3) never is to always
(4) occasionally is to often

SELDOM is to FREQUENTLY is a type 21 relationship—that is, an antonym. These two words have opposite meanings. However, (2), (3) and (4) also are antonyms. Therefore, the correct answer is based on an additional relationship. *Seldom* and *frequently* are not absolutes. *Seldom* means that something can happen, but not too often. *Top* and *bottom* and *never* and *always* are absolutes. The best choice here is (4)—occasionally is to often.

HOW TO USE NUMBER SERIES

The number series problem is a very common type of math question found on IQ tests. A number series problem gives you a list of numbers that are related by some rule. You must discover the rule to determine what the next numbers in the series should be. Here's an example:

3 7 11 15 19 23 ? ?

The rule is: Add 4 to each number to get the next. So the next two numbers are 23 + 4 = 27 and 27 + 4 = 31

Of course, number series get much more complicated. The eight lessons in *Number Series* will help you develop the skills needed to analyze any number series problem.

Organization of the Course

The course consists of two parts: an introduction that explains how to solve number series problems and a program that generates number series problems for you to solve.

Cassette version: Side A of the tape cassette contains a voice introduction. Side B contains the program that generates number series problems.

You, the user, choose the level of problem difficulty desired: easy, medium, or hard. Then, a number series problem will appear on the TV monitor. You will be asked to type in the next two numbers in the series. For example:

2 4 6 8 10 12 14 16
You type the two numbers, with a comma between them.
The next two numbers are: ?
18, 20
Then press the RETURN key.

The computer tells you whether you're right or wrong. You get three chances. If you make two mistakes on a problem, the computer gives you a hint. After your third mistake, the computer tells you the answer to that problem.

Methods for Solving Number Series

Here are some methods that will help you analyze and solve number series problems:

1. Read through the problem: you may see the pattern right away.
   Example: 3 3 4 4 5 5 6 ? ?
   The next two numbers are: 6, 7

2. Calculate the difference between each pair of numbers. Look for a pattern.
   (−2) (+3) (−2) (+3)
   Examples: 5 ◆ 3 ◆ 6 ◆ 4 ◆ 7 ? ?
   The rule is subtract 2, add 3. The next two numbers are: 5, 8

   (+1) (+2) (+3) (+4)

   1 ◆ 2 ◆ 4 ◆ 7 ◆ 11 ? ?

   The amount of change increases by 1 each time.
   The next two numbers are: 11 +5 = 16, 16 + 6 = 22

3. Look for alternating series (two series combined in one).

   Example: 1 21 3 19 5 17 7 15 ? ?
   The next number in the first series is 9. The next in the second series is 13. Answer—9, 13.

4. Look for groups of numbers within a series or other patterns if the above methods don't work.
   Example: 1 2 3 11 12 13 21 22 ? ?
   Each group of 3 numbers starts 10 greater.
   Answer: 23, 31

HOW TO USE QUANTITATIVE COMPARISONS

Many of the mathematics questions found on standardized tests ask you to compare two quantities or mathematical expressions and to indicate which of the two is greater. There are four possible choices:
1. Quantity A is larger than Quantity B (A>B).
2. Quantity B is larger than Quantity A (B>A).
3. Quantity A equals Quantity B (A=B).
4. Which quantity is larger cannot be determined from the information given (Can't determine).

Quantitative Comparisons gives you practice in answering these types of questions. It also helps you review the principles that form the basis of mathematics from beginning arithmetic through elementary algebra and plane geometry.

Organization of the Course

The course contains an introduction, seven lessons, and a final test. The introduction explains how to use the course. Each lesson tests your knowledge in one or more mathematical areas:

Lesson 1 Numbers and Arithmetic
Lesson 2 Roots and Exponents
Lesson 3 Fractions
Lesson 4 Decimals and Percent
Lesson 5 Angles and Plane Geometry
Lesson 6 Algebra
Lesson 7 Graphs and Units of Measurement

Cassette version: Each side of the cassette contains the entire course. If you have difficulty loading one side, turn the cassette over and use the second side. During the introduction, while the narrator speaks, the computer loads Lesson 1. Therefore, do NOT turn off the cassette recorder until "Lesson 1" appears on the TV monitor.

Each of the problems in *Quantitative Comparisons* tests your knowledge and understanding of one or more mathematical principles. These principles are briefly described in this guide. Each description is accompanied by one or more equations that illustrate the principle.

Here is a typical problem. Which quantity is bigger?

A	**B**
54	5 + 5 + 5 + 5

1. A>B
2. B>A
3. A=B
4. Can't determine

The answer is 1—A>B. If you don't understand the problem you should turn to Explanation 17 in this booklet. It describes the mathematical principle behind the problem and presents two equations that illustrate the principle.

For every problem in the course, the number of the relevant explanation will appear on the TV monitor if you fail in three attempts to correctly answer the problem. In Lessons 2 through 7, you can also learn the number of the relevant explanation by requesting a "hint."

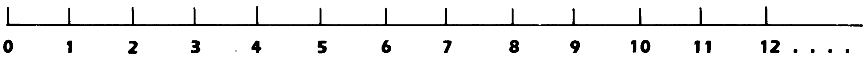
Some of the problems involve diagrams or other illustrations that are too complex to be clearly shown on the monitor. These exhibits appear in the accompanying booklet. (In PDI's *Preparing for the SAT* package, the exhibits follow the Explanations of Problems in the same booklet.)

A single pass through a lesson may not be sufficient. You may have to take a lesson two or even three times to feel confident about answering the questions.

Here is a hint to remember when answering the questions in this course: Look closely at the two values before doing any computations. In many cases, you should be able to determine which value is larger just by looking at the values. This is much faster than doing the actual computation.

Numbers

1. Numbers can be pictured by a number line:



The further to the right the number, the larger it is.

2. Additive principle: The number represented by a particular set of symbols is the sum of the numbers each symbol in the set represents.

$$1231 = 1000 + 100 + 100 + 10 + 10 + 10 + 1$$

3. Commutative laws of addition and multiplication: When two numbers are added or multiplied, the order in which they are added or multiplied is immaterial.

$$a + b = b + a \qquad a \times b = b \times a$$

$$2 + 5 = 5 + 2 \qquad 2 \times 5 = 5 \times 2$$

4. Associative laws of addition and multiplication: When more than two numbers are added or multiplied, the numbers can be grouped in any order.

$$(a + b) + c = a + (b + c) \qquad (a \times b) \times c = a \times (b \times c)$$

$$(2 + 4) + 5 = 2 + (4 + 5) \qquad (2 \times 4) \times 5 = 2 \times (4 \times 5)$$

5. Distributive law of multiplication: If a multiplicand has two or more terms, a multiplier must operate upon each of the terms in turn.

$$a(3 + b) = (a \times 3) + (a \times b)$$

$$6(3 + 4) = (6 \times 3) + (6 \times 4)$$

Arithmetic

6. "+" is a symbol indicating addition. The numbers to be added together are called *addends*. The answer is their *sum*.

$$4 + 3 = 7$$

7. The sum of any number and 0 is the number itself.

$$a + 0 = a$$

$$4 + 0 = 4$$

8. "−" is a symbol indicating subtraction. Subtraction is the opposite of addition. The *subtrahend* is subtracted from the *minuend*. The amount left over is the *remainder,* or *difference*.

$$7 - 4 = 3$$

If the subtrahend is 0, the remainder equals the minuend.

If the subtrahend equals the minuend, the remainder is 0.

9. "×" is a symbol indicating multiplication. In algebra, which uses both letters and numerals to represent numbers, multiplication is indicated by two other means:

$$4 \times 2 = 4 \cdot 2 = (4)(2)$$

Multiplication is a shorthand method of adding equal groups.

$$4 \times 2 = 2 + 2 + 2 + 2 = 8$$

The number to be multiplied is the *multiplicand.* The number by which it is multiplied is the *multiplier.* The result is the *product.*

10. The product of two numbers is zero if one of the factors is zero.

$$a \times 0 = 0$$

$$3 \times 0 = 0$$

11. "÷" is a symbol indicating division. Division is the opposite of multiplication. It splits, or divides, a group into equal parts.

$$8 \div 2 = \text{the number of times 2 goes into } 8 = 4$$

Here are two more ways to indicate division:

$$\frac{8}{2} = 8 \div 2 = 4$$

$$8/2 = 8 \div 2 = 4$$

The number being divided is the *dividend.* It is divided by the *divisor.* The result is the *quotient.*

A number divided by itself equals 1.

Square and Cube Roots

12. The square root of a number is a second number, which when multiplied by itself produces the original number.

$$\sqrt{4} = \text{the square root of } 4 = +2 \text{ or } -2$$

The symbol indicating square root is called the *radical sign.* The number under the sign is the *radical.*

Note that every positive number has two square roots.

13. A radical can be simplified by finding and removing any perfect squares.

$$\sqrt{8} = \sqrt{4 \cdot 2} = \sqrt{4}\,\sqrt{2} = 2\sqrt{2}$$

14. Like radicals can be combined by adding or subtracting the numbers in front of the radical signs. (No number in front of the sign implies "1".)

$$3\sqrt{2}+4\sqrt{2}=7\sqrt{2}$$
$$8\sqrt{3}-5\sqrt{3}=3\sqrt{3}$$

15. Two radicals can be multiplied in the usual manner.

$$(\sqrt{4})(\sqrt{9})=(\sqrt{36})$$
$$2\times 3=6$$

If numerals precede the radical sign, these should also be multiplied.

$$(2\sqrt{4})(3\sqrt{9})=6\sqrt{36}=6\times 6=36$$

16. The cube root of a number is a second number that, used as a factor three times, produces the original number.

$$\sqrt[3]{8}=\text{the cube root of } 8=2$$

Exponents

17. An exponent indicates the number of times a base is used as a factor.

$$a^3=a\times a\times a$$
$$5^3=5\times 5\times 5$$

18. A base with the exponent zero is equal to 1.

$$a^0=1$$
$$5^0=1$$

19. To multiply factors that have the same base, add their exponents.

$$a^s a^t=a^{s+t}$$
$$2^3 2^5=2^8$$

$$a^3a^0=a^{3+0}=a^3$$
$$5^3 5^0=5^{3+0}=5^3$$

20. To divide factors that have the same base, subtract their exponents.

$$a^s \div a^t = a^{s-t}$$
$$a^5 \div a^2 = a^{5-2} = a^3$$
$$2^5 \div 2^2 = 2^{5-2} = 2^3$$

If the expressions to be divided have co-efficients, divide that of the dividend by that of the divisor in the usual manner.

$$\frac{8a^s}{2a^t}=4a^{s-t}$$

21. To raise a given power by another power, multiply the two exponents.

$$(a^s)^t = a^{st}$$
$$(2^2)^3 = 2^{2\times 3} = 2^6$$

22. When a product is raised to a given power, each member of the product is raised to that power.

$$(ab)^s = a^s b^s$$

$$(5 \times 2)^2 = 5^2 \times 2^2 = 25 \times 4 = 100$$

The exponent affects only the material within the parentheses.

$$5(2)^2 = 5 \times 4 = 20$$

$$b(cd)^3 = b \times c^3 \times d^3$$

23. When a quotient is raised to a given power, each member of the quotient is raised to that power.

$$\left(\frac{a}{b}\right)^s = \frac{a^s}{b^s}$$

$$\left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2} = \frac{4}{9}$$

24. Negative exponents: A base with a negative exponent is equal to the reciprocal of the base with the corresponding positive exponent.

$$2^{-3} = \frac{1}{2^3}$$

Negative exponents follow the rules for exponents.

$$a^{-4}a^2 = a^{-4+2} = a^{-2} = \frac{1}{a^2}$$

$$5^{-4} \times 5^2 = 5^{-4+2} = 5^{-2} = \frac{1}{5^2} = \frac{1}{25}$$

25. Fractional exponents indicate radicals.

$$a^{1/2} = \sqrt{a}$$

$$a^{1/4} = \sqrt[4]{a}$$

Fractional exponents follow the rules for exponents.

$$5^2 \times 5^{3/4} = 5^{2+3/4} = 5^{11/4} = \sqrt[4]{5^{11}}$$

26. Very large and very small numbers can be expressed as powers of ten. A positive exponent represents the number of zeroes after 1.

$$1{,}000 = 10^3$$

$$1{,}000{,}000 = 10^6$$

$$8{,}450{,}000 = 8.45 \times 10^6$$

A negative exponent represents the number of digits after the decimal point.

$$10^{-5}=\frac{1}{10}\times\frac{1}{10}\times\frac{1}{10}\times\frac{1}{10}\times\frac{1}{10}=.00001$$

27. A fraction represents an indicated division.

$$\frac{4}{5} \text{ is also } 4 \div 5$$

$$\frac{\frac{1}{2}}{\frac{1}{3}}=\frac{1}{2}\div\frac{1}{3}$$

The two numerals in a fraction are called *terms.* The numeral above the *bar* is the *numerator.* The numeral below the bar is the *denominator.*

28. *Improper fractions* are fractions in which the numerator is larger than the denominator. Their value is always greater than 1.

$$\frac{5}{4}=\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=1\frac{1}{4}$$

29. A *mixed number* is the sum of a whole number and a fraction.

$$2\frac{1}{5}=2+\frac{1}{5}$$

To change a mixed number to an improper fraction, multiply the whole number by the denominator of the fraction, then add the numerator.

$$2\frac{1}{5}=(2\times5)+\frac{1}{5}=\frac{10}{5}+\frac{1}{5}=\frac{11}{5}$$

To change an improper fraction to a mixed number, divide the numerator by the denominator.

$$\frac{11}{3}=11\div3=3\frac{2}{3}$$

30. The numerator and denominator of a fraction may be multiplied or divided by the same number (other than zero) without changing the value of the fraction.

$$\frac{a}{b}=\frac{a}{b}\times\frac{c}{c}=\frac{ac}{bc} \qquad \frac{a}{b}=\frac{a}{b}\div\frac{c}{c}=\frac{ac}{bc}$$

$$\frac{1}{3}=\frac{1}{3}\times\frac{2}{2}=\frac{2}{6} \qquad \frac{2}{6}=\frac{2}{6}\div\frac{2}{2}=\frac{1}{3}$$

$\frac{1}{3}$ and $\frac{2}{6}$ are *equivalent fractions.* Their values are equal.

31. To add fractions with the same denominator, add the numerators. The denominator does not change.

$$\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$$

To add fractions with different denominators, first change the fractions so that the denominators are the same.

$$\frac{1}{3}+\frac{1}{2}=\frac{2}{6}+\frac{3}{6}=\frac{5}{6}$$

32. To subtract one fraction from another, the denominators must be the same. If they aren't, first change the fractions so that the denominators are the same. Then subtract the numerator of the subtrahend from the numerator of the minuend. The denominator stays the same.

$$\frac{7}{8}-\frac{3}{8}=\frac{4}{8}$$

$$\frac{5}{6}-\frac{1}{4}=\frac{10}{12}-\frac{3}{12}=\frac{7}{12}$$

33. To multiply two fractions, multiply one numerator by the other to obtain the numerator of the product. Then multiply one denominator by the other to obtain the denominator of the product.

$$\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$$

$$\frac{3}{4}\times\frac{2}{5}=\frac{6}{20}$$

34. To multiply a fraction by a whole number, multiply the numerator by the whole number. Keep the same denominator.

$$a\times\frac{b}{c}=\frac{ab}{c}$$

$$6\times\frac{2}{15}=\frac{12}{15}$$

35. To multiply a mixed number by a fraction or by a whole number, first change the mixed number to an improper fraction.

$$2\frac{1}{3}\times\frac{2}{5}=\frac{7}{3}\times\frac{2}{5}=\frac{14}{15}$$

$$2\frac{1}{3}\times 2=\frac{7}{3}\times 2=\frac{14}{3}$$

36. To divide a fraction by another fraction, invert the divisor. Then multiply.

$$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}=\frac{ad}{bc}$$

$$\frac{2}{3}\div\frac{1}{4}=\frac{2}{3}\times\frac{4}{1}=\frac{8}{3}$$

37. To divide a fraction by a whole number, think of the divisor as a fraction with a denominator of 1 ($2=\frac{2}{1}$). Invert the divisor. Then multiply.

$$\frac{6}{7}\div 2=\frac{6}{7}\div\frac{2}{1}=\frac{6}{7}\times\frac{1}{2}=\frac{6}{14}$$

38. To divide a whole number by a fraction, invert the divisor. Then multiply. (Think of the dividend as a fraction.)

$$2\div\frac{6}{7}=\frac{2}{1}\times\frac{7}{6}=\frac{14}{6}$$

39. To divide a mixed number by a fraction, first change the mixed number to an improper fraction.

$$1\frac{3}{5}\div\frac{2}{7}=\frac{8}{5}\div\frac{2}{7}=\frac{8}{5}\times\frac{7}{2}=\frac{56}{10}$$

40. If arithmetic operations are indicated in either the numerator or the denominator, do these operations first.

$$\frac{(6+2)(3\cdot 5)}{(8\cdot 3)+6}=\frac{(8)(15)}{(24)+6}=\frac{120}{30}$$

Decimals and Percent

41. A decimal is a fraction in which the denominator is a multiple of 10.

$$.2=\frac{2}{10}$$

$$3.07=3\frac{7}{100}$$

Numerals to the right of the decimal point go down in value as their position from the decimal point increases. Numerals to the left of the decimal point go up in value as their position from the decimal point increases.

.2 is 10 times greater than .02

20. is 10 times greater than 2.

42. The value of a decimal fraction is not changed by adding zeroes at the end of the numeral.

$$.4=.40=.400$$

43. To add or subtract decimal fractions, the decimal points must be placed one above the other.

$$\begin{array}{r} .5 \\ 2.9 \\ +10.02 \\ \hline 13.42 \end{array} \qquad \begin{array}{r} 6.2 \\ -\ .05 \\ \hline 6.15 \end{array}$$

44. To multiply decimal fractions, multiply the numbers. Then count the number of figures to the right of the decimal point in the multiplicand and the multiplier. This total equals the number of figures to the right of the decimal point in the product.

$$\begin{array}{r} 6.2 \\ \underline{.8} \\ 4.96 \end{array} \qquad \begin{array}{r} .005 \\ \underline{2} \\ .010 \end{array}$$

45. To divide decimal fractions, make the divisor a whole number by moving the decimal point the necessary number of places to the right. Move the decimal point of the dividend an equal number of places to the right. Place the decimal point in the quotient directly above the decimal point in the dividend.

$$1.2 \div .5 = 12.0 \div 5 = 2.4$$

46. A percent is a fraction with a denominator of 100.

$$5\% = \frac{5}{100}$$

$$12.83\% = \frac{12.83}{100}$$

$$6\frac{1}{2}\% = \frac{6.5}{100}$$

47. To write a percent as a decimal, drop the percent symbol and move the decimal point two places to the left.

$$33\% = .33$$

To write a fraction as a percent, change the fraction to a decimal. Move the decimal point two places to the right. Add the percent symbol.

$$\frac{3}{4} = .75 = 75\%$$

48. To multiply a numeral by a percent, change the percent to a decimal or to a fraction. Then multiply.

$$25\% \text{ of } 40 = \frac{1}{4} \times 40 = \frac{40}{4} = 10$$

$$25\% \text{ of } 40 = .25 \times 40 = 10.00$$

The following formula is used to calculate interest:

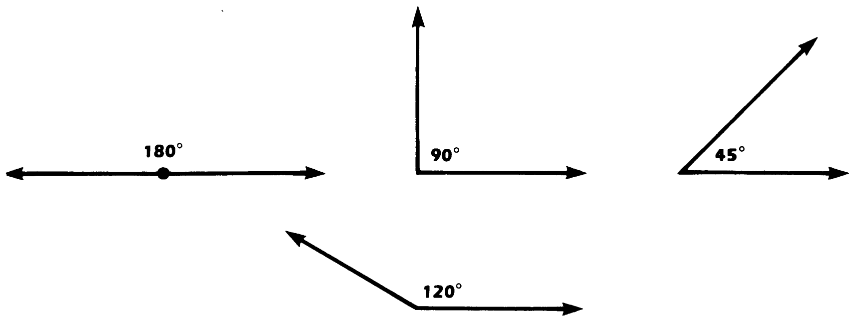
I = PRT

Interest = Principle × Rate × Time

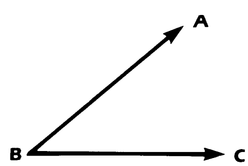
2% of \$1000 for 1 year = 1000 × .02 × 1 = \$20

$$2\% \text{ of } \$1000 \text{ for 6 months} = 1000 \times .02 \times \frac{1}{2} = \$10$$

49. The size of an angle can be measured in degrees. A *straight angle* has 180°. A 90° angle is called a *right angle*. An angle containing less than 90° is an *acute angle*. An angle containing between 90° and 180° is an *obtuse angle*.

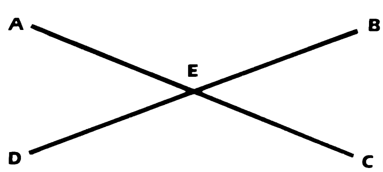


The point at which the two line segments meet is the *vertex* of the angle.



The angle is named $\measuredangle ABC$ ($\measuredangle$ is the symbol for angle). B is the vertex of $\measuredangle ABC$.

50. When two lines cross, the opposite angles are equal.



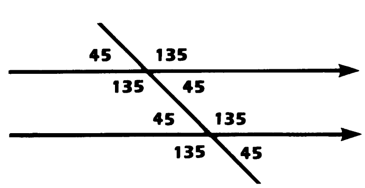
$\measuredangle AED = \measuredangle BEC$

$\measuredangle AEB = \measuredangle DEC$

The angles on the same side of a line are supplementary. That is, they equal 180°.

$$\measuredangle AEB + \measuredangle BEC = \measuredangle AED + \measuredangle DEC = 180°$$

51. Corresponding angles formed when a transversal crosses two parallel lines are equal.

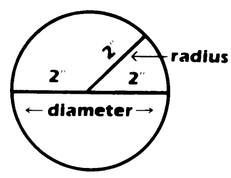


The interior angles on the same side of the transversal are supplementary (that is, they equal 180°).

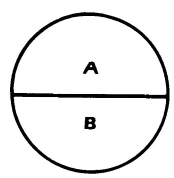
Alternate interior angles are equal.

Circles

52. A line from the center of a circle to the outside is called a *radius*. All the radii of a circle have the same length. A *diameter*, a line passing through the center of a circle, consists of two radii.

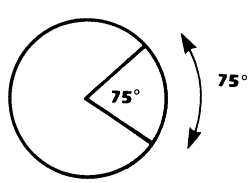


53. A diameter cuts a circle in half.

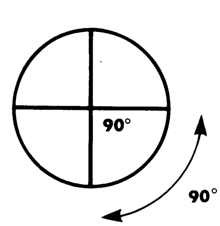


Area A = Area B

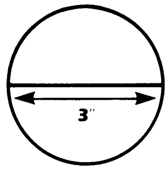
54. An angle at the center of a circle has the same number of degrees as the arc that it cuts off on the circle. The total number of degrees in a circle is 360.



55. If two diameters are perpendicular to one another, they form right angles, each having 90°. Each arc cut off on the circle also contains 90°.

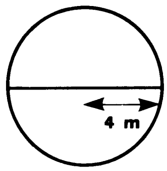


56. The circumference of a circle equals the diameter multiplied by pi (3.1416).



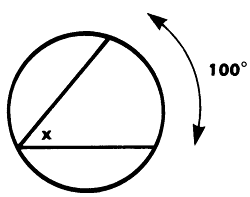
$C = \pi d = (3.1416)(3) = 9.4248''$

57. The area of a circle is πr^2.



$A = \pi r^2 = (3.1416)(4^2) = (3.1416)(16) = 50.2656\ m^2$

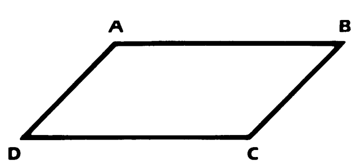
58. The degrees in an inscribed angle equals $\frac{1}{2}$ the degrees in its intercepted arc.



$x = \frac{1}{2}(100°) = 50°$

Parallelograms

59. A parallelogram consists of two pairs of parallel lines. Rectangles, squares, and rhomboids are parallelograms. The opposite sides of a parallelogram are equal.



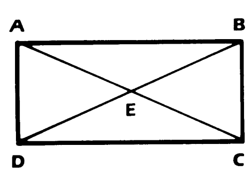
AB is parallel to DC

AD is parallel to BC

AB = DC

AD = BC

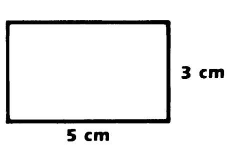
60. In a parallelogram, the diagonals bisect each other.



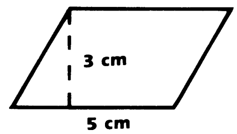
AE = EC

DE = EB

61. The area of a parallelogram equals the height times the base.

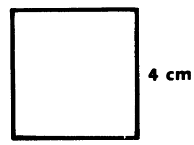


$Area = h \times b = 3\ cm \times 5\ cm = 15\ cm^2$



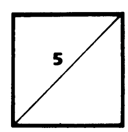
Area = h × b = 3 cm × 5 cm = 15 cm^2

62. The area of a square is the length of one side times itself.



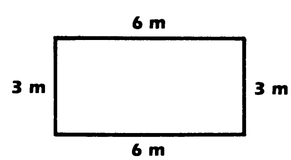
Area = $S^2 = (4\text{ cm})^2 = 16\text{ cm}^2$

63. The area of a square is equal to ½ the square of its diagonal.



If d = 5 m then $A = \frac{d^2}{2} = \frac{5^2}{2} = \frac{25}{2} = 12.5\text{ m}^2$

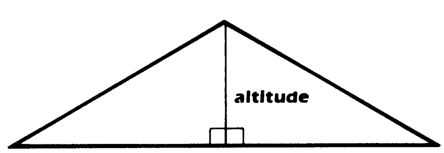
64. The perimeter, or distance around the outside, of a parallelogram equals the sum of the lengths of all four sides.



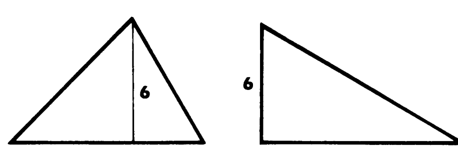
P = 6 m + 3 m + 6 m + 3 m = 18 m

Triangles

65. The perpendicular distance from one side of a triangle to the vertex of the angle opposite that side is called an *altitude.*

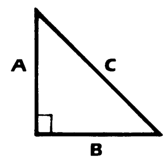


66. Triangles with equal bases and heights are *equal triangles.* Regardless of their shapes, they have the same area.



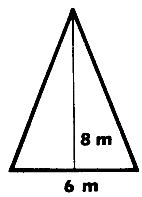
These are equal triangles.

67. In a right triangle, the sum of the squares of the sides equals the square of the hypotenuse. This is the Pythagorean Theorem.



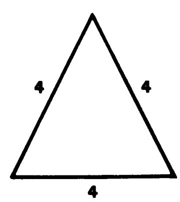
$A^2 + B^2 = C^2$

68. To determine the area of a triangle, multiply the length of one side times the altitude of that side. Divide the result by 2.



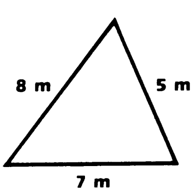
$$\text{Area} = \frac{6\ \text{m} \times 8\ \text{m}}{2} = \frac{48\ \text{m}^2}{2} = 24\ \text{m}^2$$

69. A triangle with three equal sides is called an *equilateral triangle*. Its area equals the square of one side divided by 2.



$$A = \frac{s^2}{2} = \frac{4^2}{2} = \frac{16}{2} = 8$$

70. The perimeter of a triangle equals the sum of the lengths of the three sides.



$$P = 8\ \text{m} + 7\ \text{m} + 5\ \text{m} = 20\ \text{m}$$

Algebraic Equations

71. The addition, subtraction, multiplication, or division of both sides of an equation by the same quantity produces a new equation with the same roots.

$a+5=7$	$6c=18$	$\frac{d}{4}=3$
$a+5-5=7-5$	$\frac{6c}{6}=\frac{18}{6}$	$\frac{d}{4}\times 4=3\times 4$
$a=2$	$c=3$	$d=12$

$b-3=7$

$b-3+3=7+3$

$b=10$

Signed Numbers

72. Signed numbers are numbers preceded by a postive or negative sign (any number not preceded by such a sign is presumed to be positive). Signed numbers can be pictured by a number line.

. . . −7 −6 −5 −4 −3 −2 −1 0 +1 +2 +3 +4 +5 +6 +7 +8 +9 . . .

The further to the right the number, the greater its value.

73. Adding signed numbers: If the signs are the same, add the numbers in the usual manner; the sum has the same sign as the addends.

$$(+3)+(+4)=+7$$

$$(-3)+(-4)=-7$$

If the signs are opposite, the sum is the difference between the numbers preceded by the sign of the larger number.

$$(+3)+(-4)=-1$$

74. Subtracting signed numbers: Change the sign of the subtrahend and add the numbers.

$$(+a)-(+b)=(+a)+(-b)$$

$$(+4)-(+3)=(+4)+(-3)=1$$

$$(+a)-(-b)=(+a)+(+b)$$

$$(+4)-(-3)=(+4)+(+3)=7$$

75. Multiplying and dividing signed numbers: If the signs are alike, the result is a positive number. If the signs are different, the result is a negative number.

$(a)(b)=+ab$	$10a \div 5a = 2$
$(3)(4)=+12$	$(-10a) \div 5a = -2$
$(+a)(-b)=-ab$	$(-16) \div (-8) = 2$
$(3)(-4)=-12$	$(16) \div (-8) = -2$

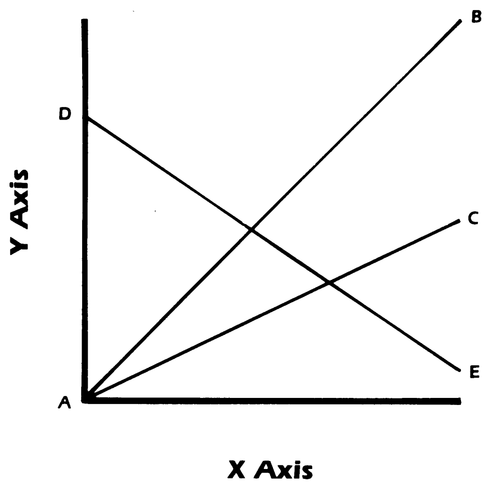
76. Signed fractions: If any two of the three signs of a fraction are changed, the value of the fraction will remain the same.

$$+\frac{+a}{+b} = +\frac{-a}{-b} = -\frac{+a}{-b} = -\frac{-a}{+b}$$

$$\frac{3}{5}=\frac{-3}{-5}=-\frac{+a}{-5}=-\frac{-3}{+5}$$

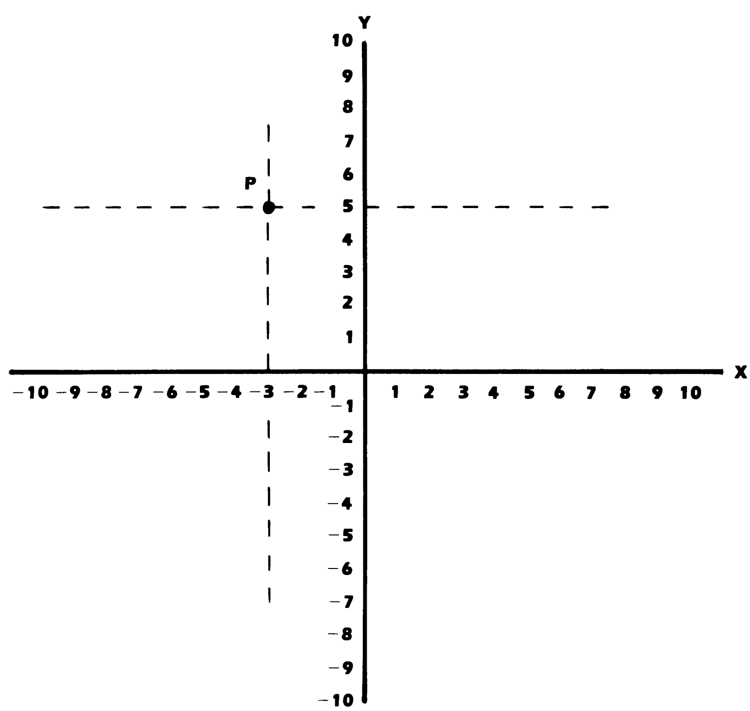
Graphs

77. The horizontal line of a graph is called the *x axis*. The vertical line is the *y axis*. The value of x increases to the right of the intersection of the two axes. The value of y increases upward from the intersection.



78. The *slope* is the steepness of the rise of a line. AB (above) has a steeper slope than AC. DE has a *negative* slope. A line parallel to the x axis has a 0 slope. A line parallel to the y axis has an *infinite* slope.

79. In analytic geometry, two intersecting straight lines are called *coordinate axes*. The point at which they intersect is called the *origin*.



Generally, as shown here, the units on the two axes are the same. The positive x axis is on the right; the negative x axis is on the left. The positive y axis is above the origin; the negative y axis is below.

The x coordinate of a point, P, is found by drawing a line parallel to the y axis. The y coordinate of P is found by drawing a line parallel to the x axis. The coordinates are written like this:

P= (x,y)

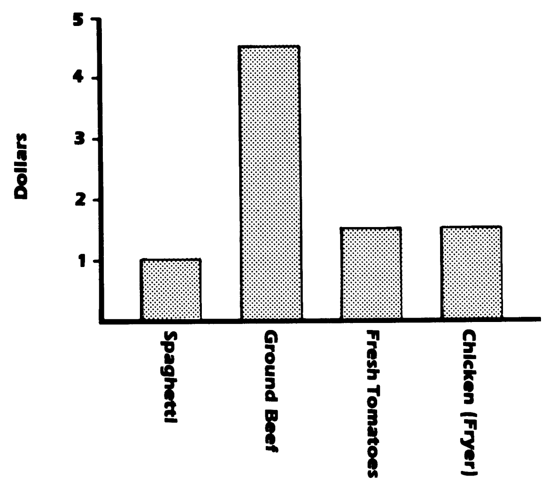
P= (−3, +5)

The x coordinate is always written first.

80. A *graph* is a picture. It illustrates numerical information. There are three main types of graphs: bar, line, and circle.

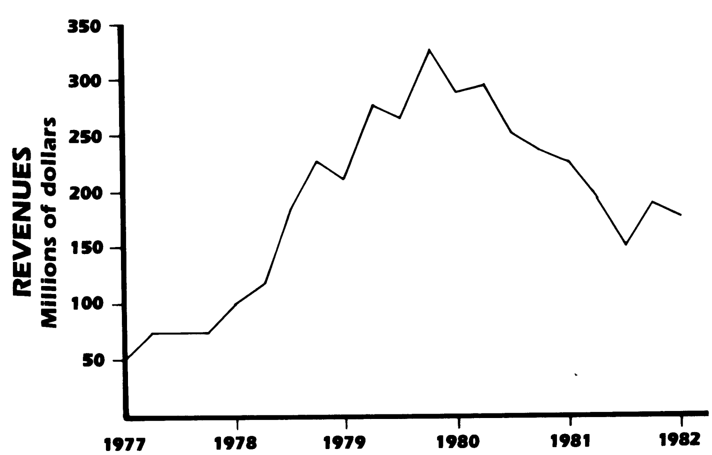
81. A *bar graph* uses bars to represent numbers. On a horizontal bar graph, the bars are horizontal. Values are measured along the horizontal axis. On a vertical bar graph, the bars are vertical. Values are measured along the vertical axis.

This is a vertical bar graph. It shows that a kilo of spaghetti costs $1.00, while a kilo of ground beef costs $4.60. The cost of tomatoes and chicken are the same ($1.79).



Food Costs (per kg)

82. A *line graph* consists of a series of points connected by line segments.

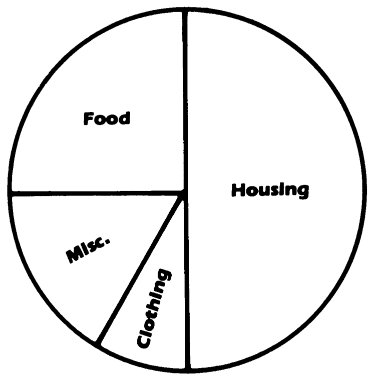


Growth at Xyp Corp.

Here, for example, you see that revenues grew from $50 million in the first quarter of 1977 to $100 million in the first quarter of 1978. After 1979, revenues generally decreased.

83. A *circle graph* is sometimes called a pie chart. It is used to illustrate parts, or percentages, of a total amount.

Student's budget for March



Item	Amount	% of Total
Housing	$200	50
Food	$100	25
Clothing	$32	8
Misc.	$68	17
Total	$400	100

Measurement

84. Linear Measurement
10 millimeters (mm) = 1 centimeter (cm)
10 centimeters = 1 decimeter (dm)
10 decimeters = 1 meter (m)
10 meters = 1 dekameter (dkm)
10 dekameters = 1 hectometer (hm)
10 hectometers = 1 kilometer (km)

85. Measurement of Weight
10 milligrams (mg) = 1 centigram (cg)
10 centigrams = 1 decigram (dg)
10 decigrams = 1 gram (g)
10 grams = 1 dekagram (dkg)
10 dekagrams = 1 hectogram (hg)
10 hectograms = 1 kilogram (kg)
1000 kilograms = 1 metric ton

86. Measurement of Capacity
10 milliliters (ml) = 1 centiliter (cl)
10 centiliters = 1 deciliter (dl)
10 deciliters = 1 liter (l)
10 liters = 1 dekaliter (dkl)
10 dekaliters = 1 hectoliter (hl)
10 hectoliters = 1 kiloliter (kl)

87. Measurement of Time
60 seconds = 1 minute
60 minutes = 1 hour
24 hours = 1 day
7 days = 1 week
365 days = 1 year
52 weeks = 1 year
12 months = 1 year

EXHIBITS

The following diagrams are needed to answer some of the questions in the course *Quantitative Comparisons*.

Exhibit #1

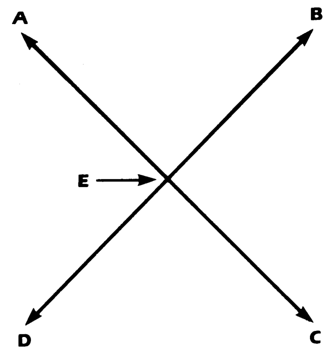


Exhibit #2

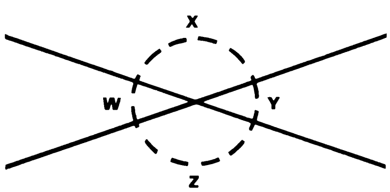


Exhibit #3

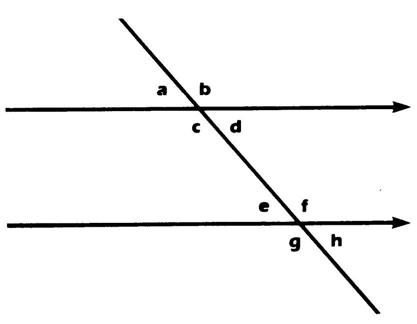


Exhibit #4

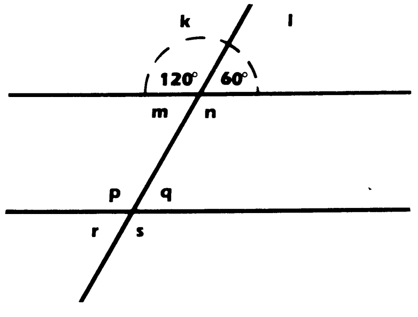


Exhibit #5

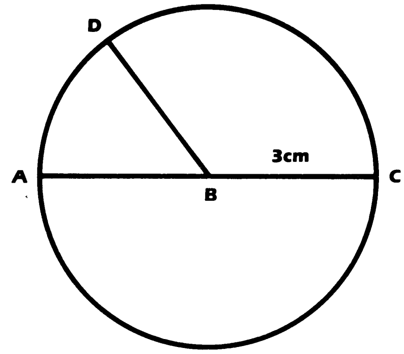


Exhibit #6

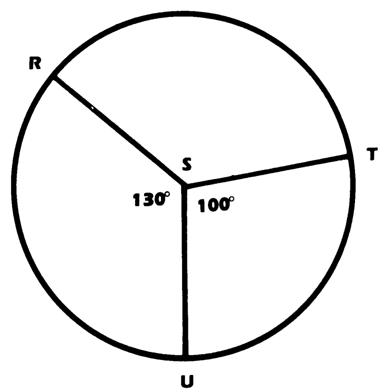


Exhibit #7

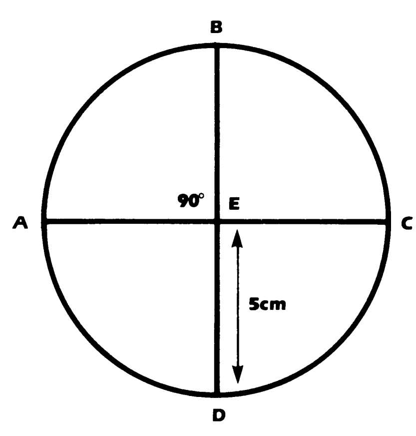
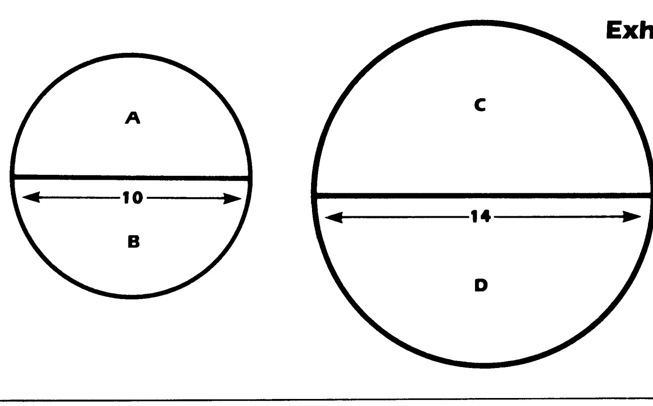


Exhibit #8



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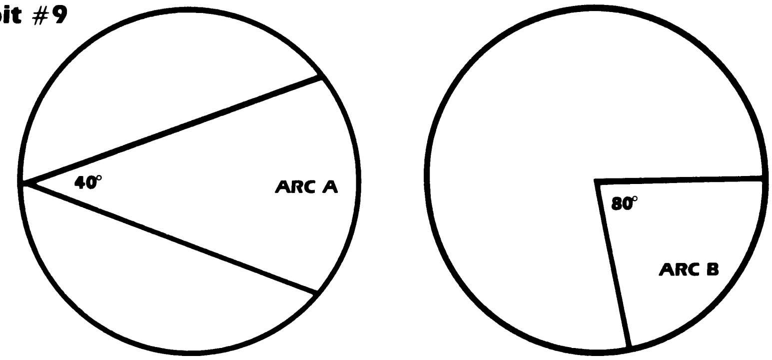

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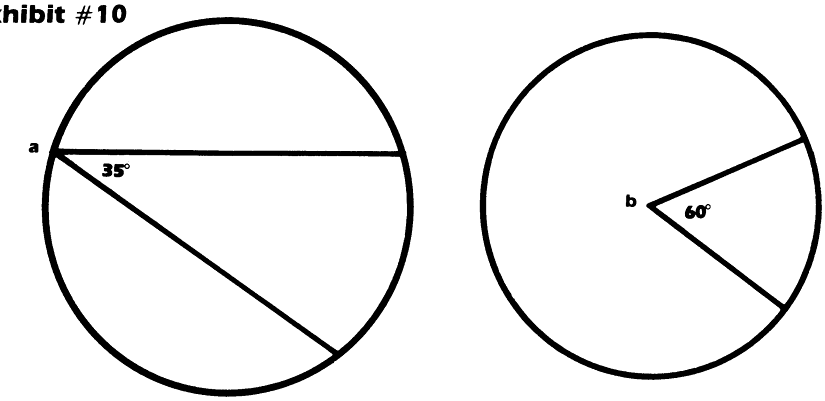

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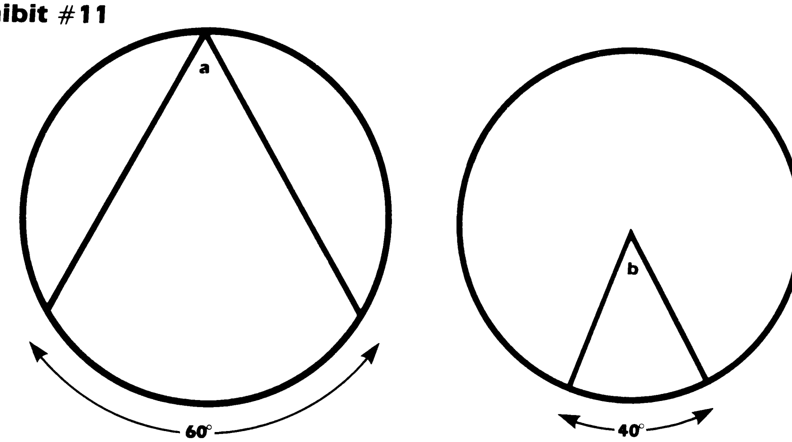

A	B

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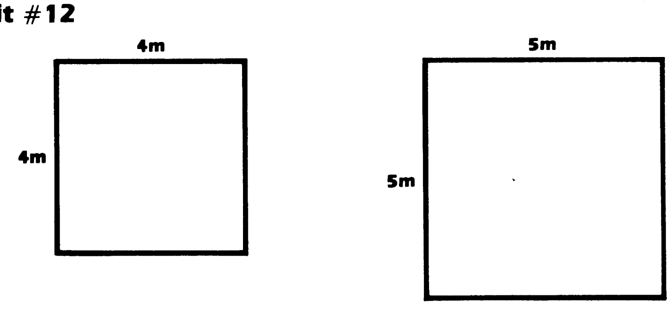

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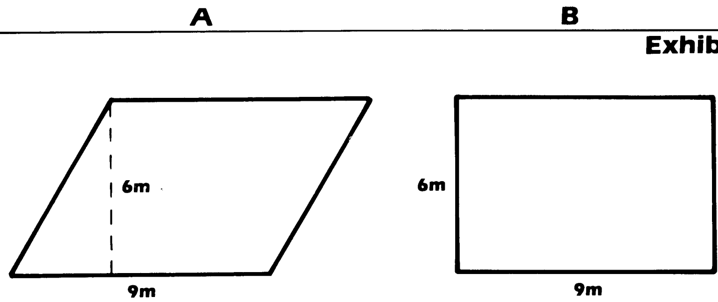

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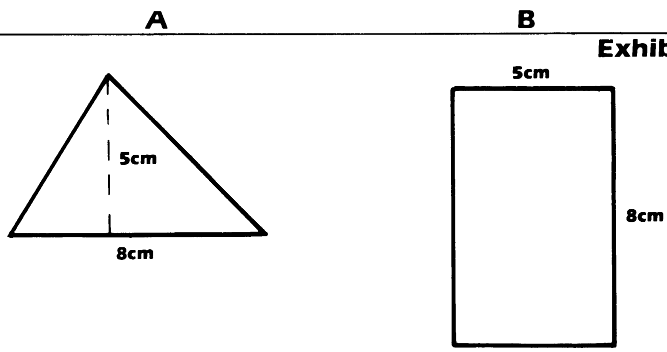

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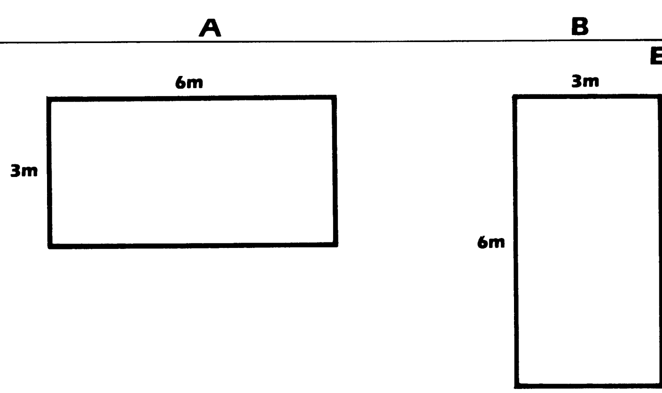

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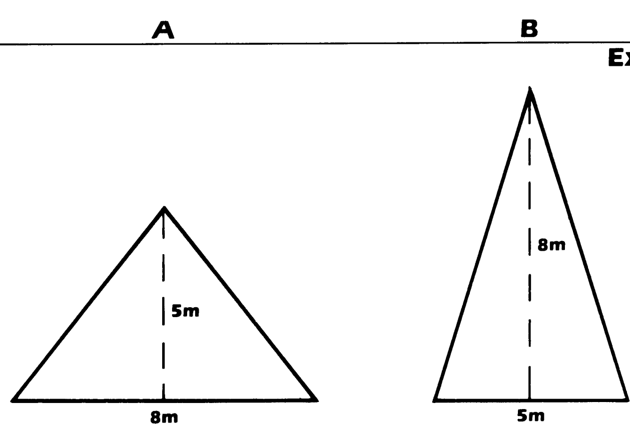
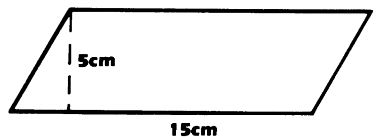
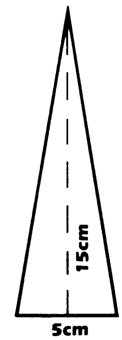


Exhibit #17

A



B



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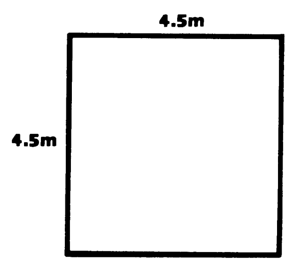
A



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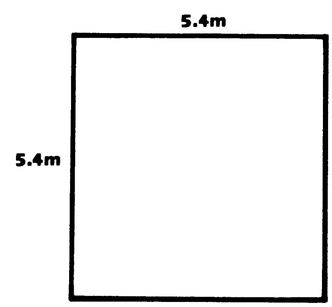

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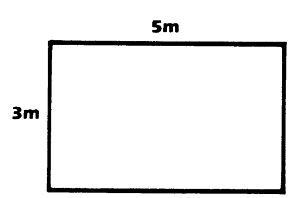
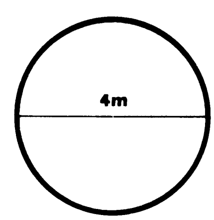
A



B



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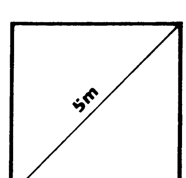
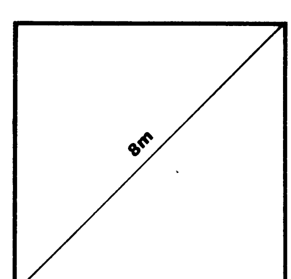
A

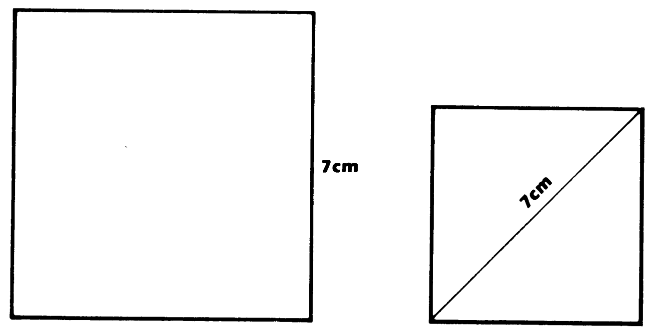


B



A B

Exhibit #21

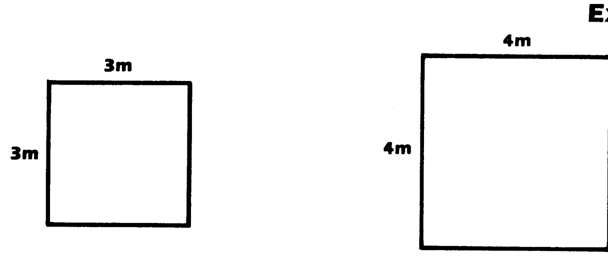


A B

Exhibit #22

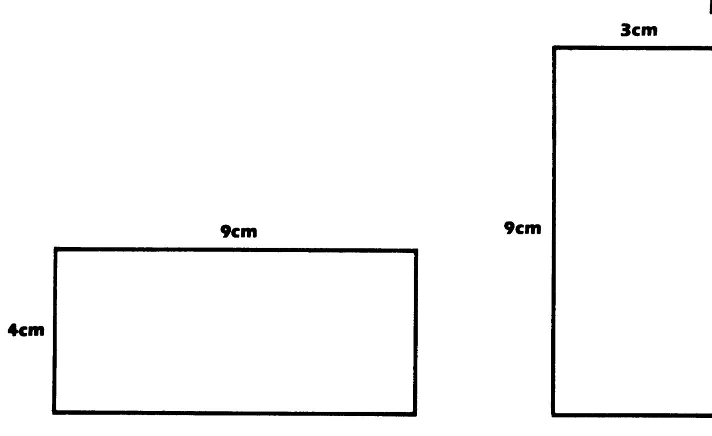
A B

Exhibit #23



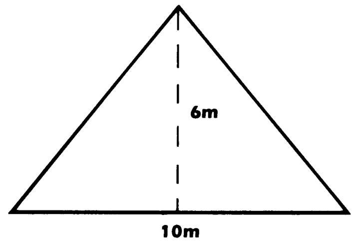
A B

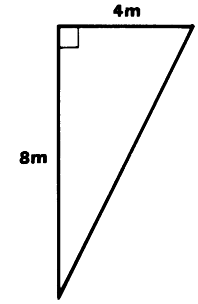
Exhibit #24



A B

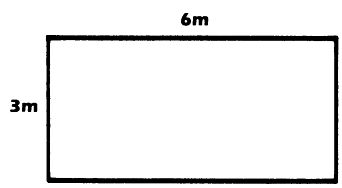
Exhibit #25

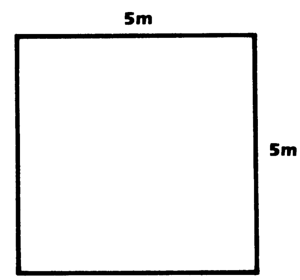




A B

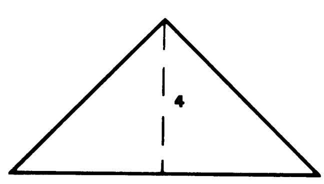
Exhibit #26

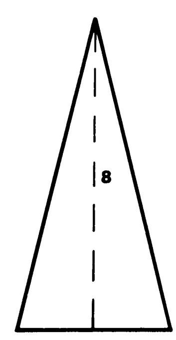




A B

Exhibit #27





A B

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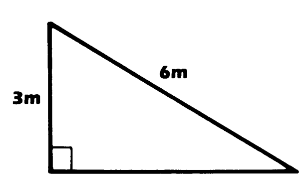

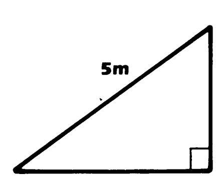

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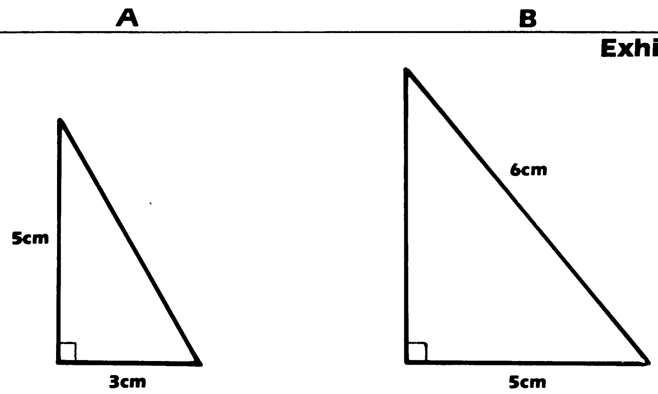

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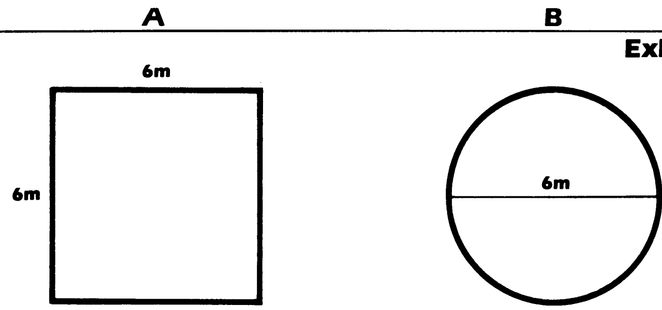

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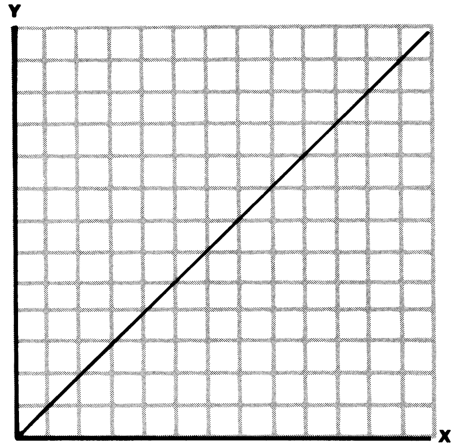

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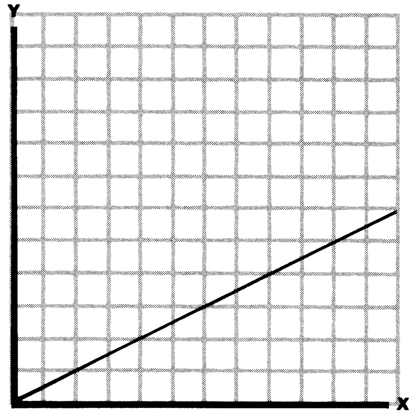

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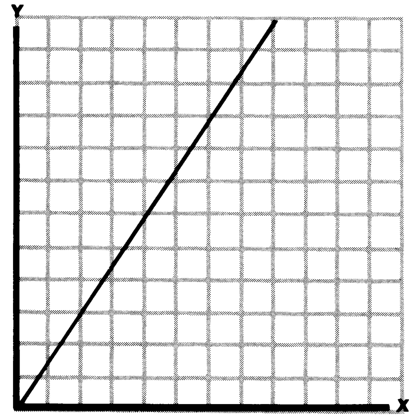

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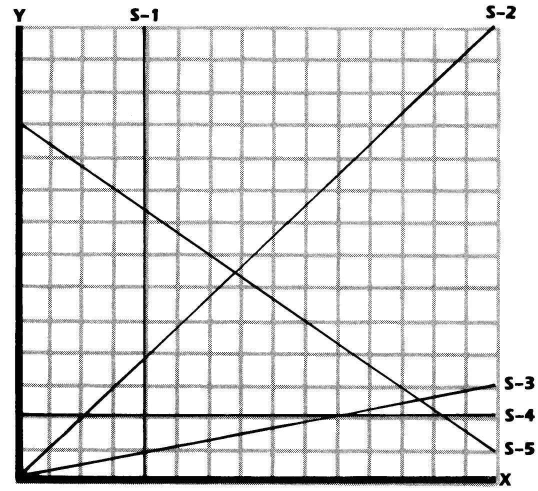

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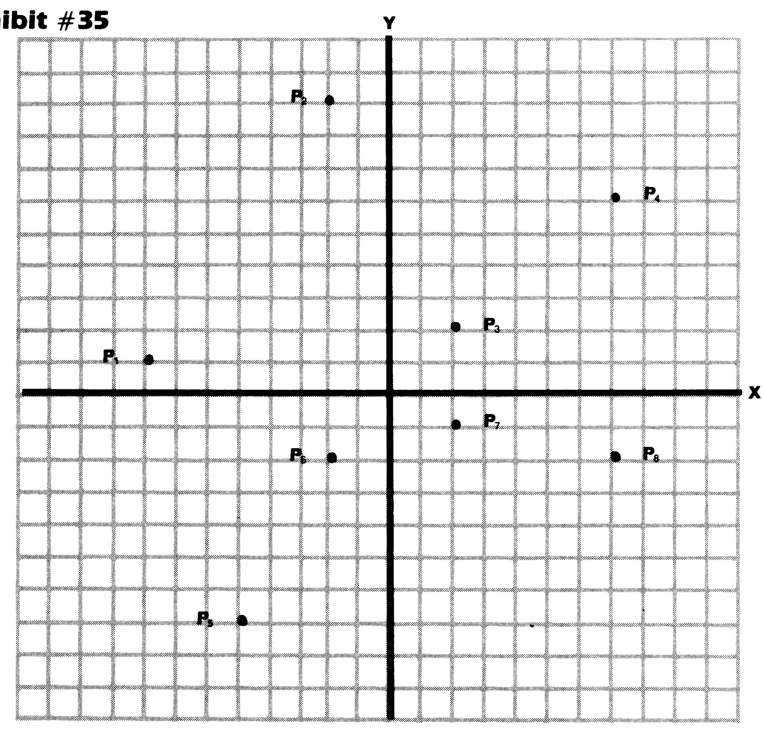

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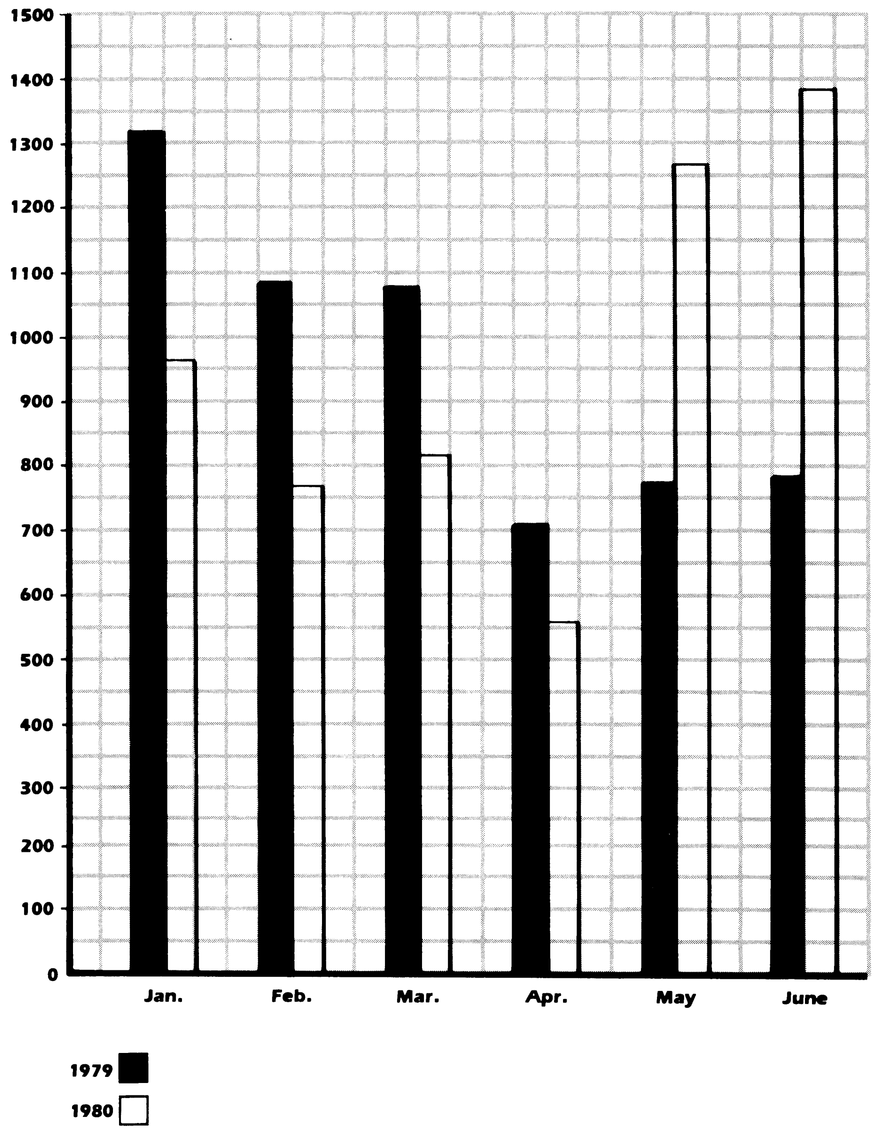
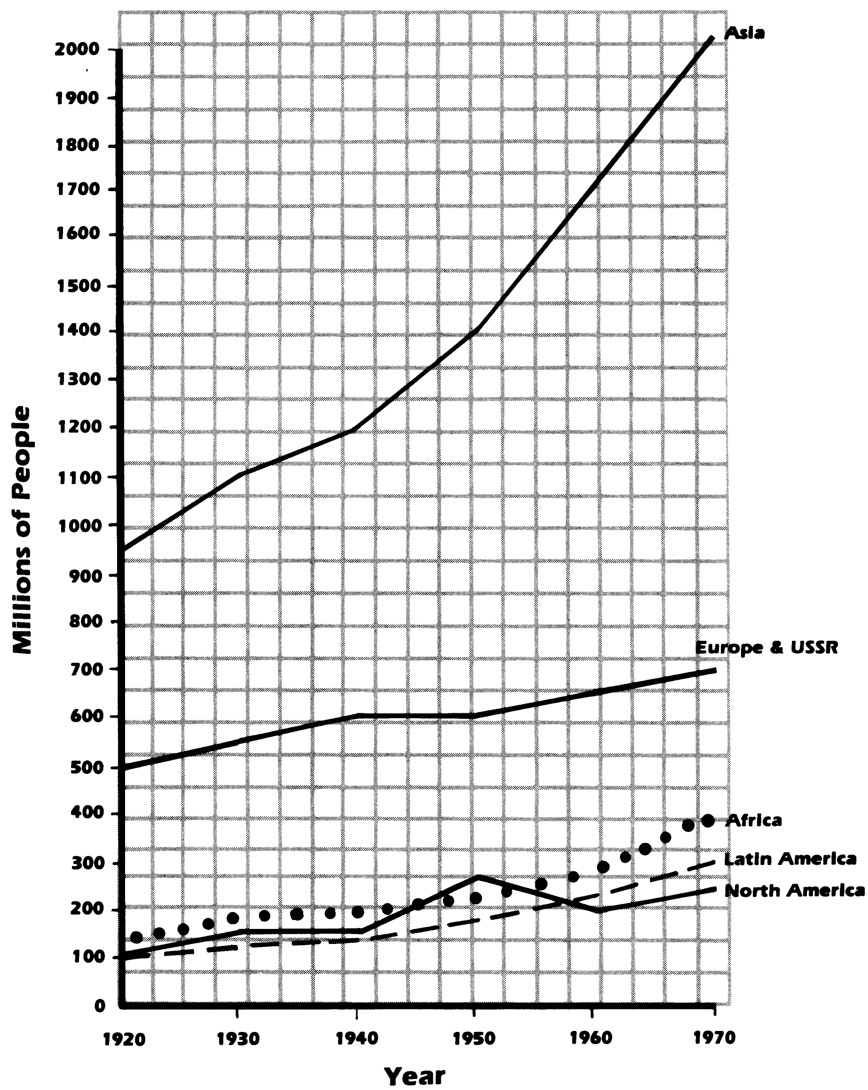


Felonies Committed on Subways in New York City

Exhibit #37



POPULATION OF THE WORLD

Exhibit #38

The World Export Market

Percentage share of total world exports

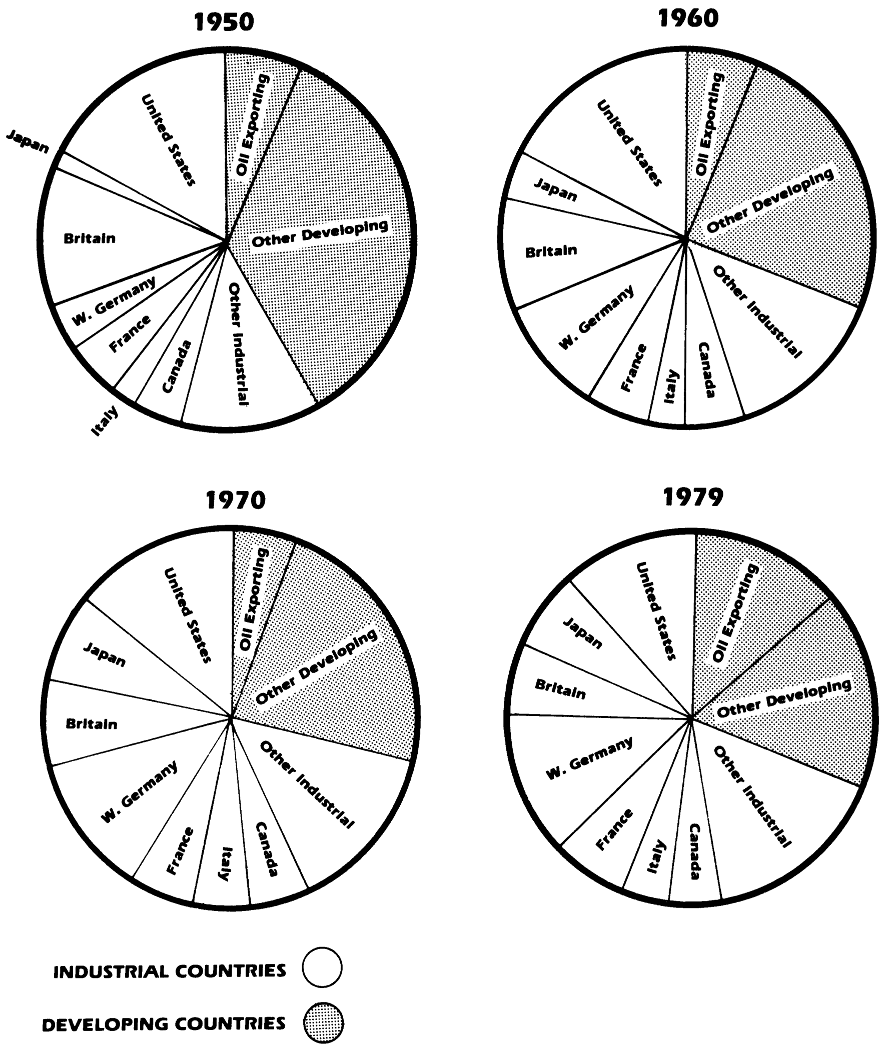
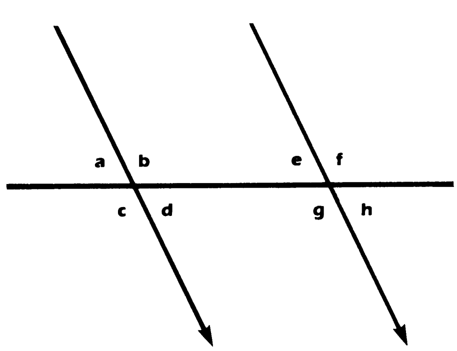


Exhibit #39



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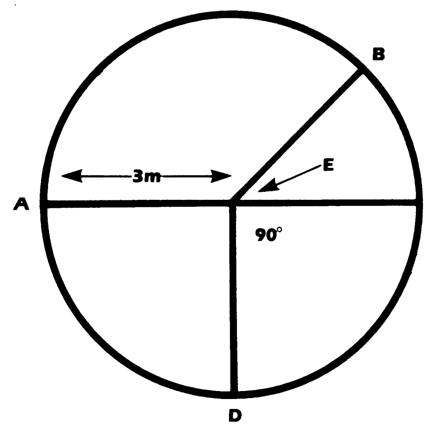
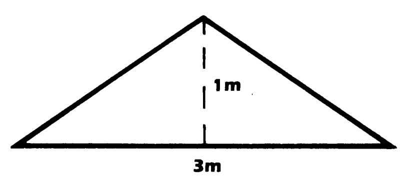
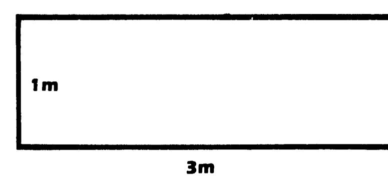


A	B

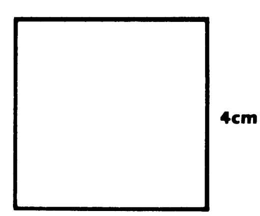
Exhibit #41

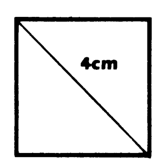




A	B

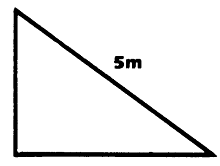
Exhibit #42

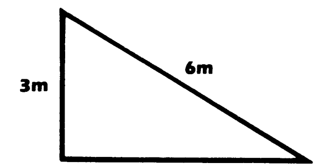




A	B

Exhibit #43





NOTES

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